ACOUSTICAL IMPEDANCE OF FOG

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ABSTRACT. The acoustical impedance of porous bodies has been related to their porosity, and flow resistance, and it is possible to evaluate it when their physical properties are known. In the present note the acoustical impedance of air containing water particles in suspension has been calculated in a straight forward manner directly from the hydrodynamical laws. The resulting formula is practically similar to that for solids.

Sewell (1910) calculated the amount of energy scattered by water particles ; he found that the fraction of energy scattered is very small. The present author has utilised some of his results, specially the formula which determines amplitude of vibration of the water particles with respect to the vibrating air particles.

The hydrodynamical resistance to oscillatory air flow against smooth water particles is given by (Lamb, 1930)

$$R = n\frac{4\pi}{3}\rho_0 a^3\left[\frac{1}{2}+\frac{9}{4a}\sqrt{\frac{2\nu}{\sigma}}\right]\frac{dU}{dt}+n3\pi\rho_0 a^3\sigma\left[\frac{1}{a}\sqrt{\frac{2\nu}{\sigma}}+\frac{1}{a^2}\frac{2\nu}{\sigma}\right]U.$$

where a = radius of solid particles, ρ_0 = density of air, $\sigma = 2\pi$ (frequency), ν = kinematic viscosity, n = number of particles per c.c.

In evaluating the resistance due to a large number of particles, the diffraction of sound by the neighbouring particles has been neglected. If $U \propto e^{j\sigma t}$ then $R = R_0 + jX$ where

$$\left.\begin{aligned} R_0 &= 6\pi\rho_0 a\nu n\left(1+a\sqrt{\frac{\sigma}{2\nu}}\right)\\ X &= \frac{4\pi}{3}\rho_0 a^3 n\sigma\left(\tfrac{1}{2}+\frac{9}{4a}\sqrt{\frac{2\nu}{\sigma}}\right)\end{aligned}\right\} \qquad \ldots \quad (1)$$

When the air flow is not oscillatory, X is zero, and we have

$$R'_0 = 6\pi a n\eta$$

which is Stokes' law. The following table gives the values of R_0/ρ_0 and X/ρ_0 for different frequencies

$$R_0 = 6\nu\rho_0 an(1+f),\ f = a\sqrt{\pi N/{\cdot}132}$$

$$a = 10^{-3}\text{cm.};\ n = 10^6,\ \nu = {\cdot}132$$

$$f = 4.84\times 10^{-3}\sqrt{N}$$

N = frequency.

TABLE I

N	f	R_0/ρ_0	X_0/ρ_0
128	.054	2.48×10^3	1.28×10^2
256	.075	2.53×10^3	1.94×10^2
512	.105	2.66×10^3	2.60×10^2
1024	.140	2.76×10^3	3.80×10^2
2048	.196	2.90×10^3	5.55×10^2

It will be noticed that R_0 is much larger than X_0; and that f affects R_0 at very high frequencies only. Consequently if we replace R_0 by the flow resistance R'_0 much error will not be produced in the case of small particles of the order of 10^{-3} cm. When the particles increase in size, f becomes appreciable, and the flow resistance R'_0 will be much smaller than R_0.

Equation of continuity.—Since there are n particles per unit volume, they occupy a volume β and, therefore, the volume available is, $(1-\beta)$,

where
$$\beta = \frac{4}{3}\pi a^3 n$$

Considering the flow of air through planes perpendicular to the x axis at x and $x+\Delta x$ we obtain

$$-\frac{d\rho u}{dx}\Delta x = \frac{d\rho}{dt}(1-\beta)\Delta x \qquad \ldots \quad (2)$$

since the water particles are not compressible, u being the particle velocity of air. Writing $P=(1-\beta)$ and designating it as porosity we re-write the above equation as

$$-\frac{du}{dx} = -\frac{P}{\rho_0}\frac{d\rho}{dt}$$

If we assume the gas law $P_1\rho^{-\gamma}$ = constant, γ being the ratio of specific heats, then

$$p = \frac{\gamma P_0}{\rho_0}\Delta\rho$$

p = excess pressure, P_0, ρ_0 normal pressure and density.

In the present case if the suspensions are supposed to take part in the heat reactions, then the resulting value of γ_s is given by

$$\Gamma = \frac{PC_p + \lambda(1-P)}{PC_v + \lambda(1-P)} \qquad \ldots \quad (3)$$

where λ is the specific heat of the suspension, which in the present case is unity. When these modifications are introduced the equation (2) becomes

$$\frac{du}{dx} = -\frac{P}{\Gamma\rho_0}\frac{dp}{dt} = -\frac{Pj\sigma}{\rho_0\Gamma}p \qquad \ldots \quad (4)$$

where $p \propto e^{j\sigma t}$.

While the equation of motion is

$$-\frac{dp}{dx}\Delta x=\rho_0\Delta x\,\frac{du_1}{dt}+\rho'\Delta x\,\frac{du_2}{dt}+(R_0+jx)(u_1-u_2) \qquad \dots \quad (5)$$

where $\rho'=\frac{4}{3}\pi a^3\rho_1$, ρ_1 = density of water particles, u_1 particle velocity for air, and u_2 the same for the suspensions. The first term represents the difference of pressure between the points x and $x+\Delta x$ in the positive direction of x axis. In the right hand side $\frac{du_1}{dt}$ represents the acceleration of air particles, $\frac{du_2}{dt}$ the same for water particles of mass $\rho'\Delta x$ in volume Δx. The third term represents the resistance due to water particles, (u_1-u_2) being the relative velocity of air with respect to water particles. Sewell (*loc. cit.*) has shown that

$$u_2/u_1=(1-jq)L^2 \qquad \dots \quad (6)$$

$$q=\frac{2}{9}\frac{a^2\sigma}{\nu}\frac{\rho_1}{\rho_0};\quad L=\frac{1}{(1+q^2)^{\frac{1}{2}}},$$

ρ_1 = density of water particles. Hence

$$-\frac{dp}{dx}=u_1 j\sigma\left\{\rho_0+\rho'L^2+\frac{X(1-L^2)+R_0qL^2}{\sigma}\right\}+u_1\{R_0(1-L^2)-XqL^2\} \qquad (7)$$

$$-\frac{dp}{dx}=R_1u_1+j\sigma\rho_2u_1 \qquad \dots \quad (8)$$

$$R_1=\{R_0(1-L^2)-XqL^2\};\ \rho_2=\rho_0+\rho^1L^2+\frac{X(1-L^2)+R_0qL^2}{\sigma}$$

With the help of (4) we get

$$\frac{d^2p}{dx^2}=\frac{Pp}{C_2^2\rho_2}(j\sigma R_1-\sigma^2\rho_2),\ C_2^2=\Gamma P_0/\rho_2$$

$$i.e.,\ \frac{d^2p}{dx^2}=\frac{j^2Q^2\sigma^2P}{C_2^2}p \qquad \dots \quad (9)$$

where

$$Q=(1-jR_1/\sigma\rho_2)^{\frac{1}{2}}$$

Therefore

$$p=Ae^{-\frac{j\sigma Q\sqrt{P}}{C_2}x}e^{j\sigma t}$$

Since

$$\frac{du}{dx}=-\frac{P}{C_2^2\rho_2}\frac{dp}{dt}=-\frac{Pj\sigma}{C_2^2\rho_2}p$$

$$Z_2=\frac{p}{u}=\frac{C_2\rho_2}{\sqrt{P}}Q=\frac{C_2\rho_2}{\sqrt{P}}\left(1-\frac{jR_1}{\sigma\rho_2}\right)^{\frac{1}{2}}\dots\dots$$

and

$$\frac{Z_2}{Z_0}=\left(\frac{\Gamma\rho_2}{\gamma\rho_0}\right)^{\frac{1}{2}}\frac{1}{\sqrt{P}}\left(1-\frac{jR_1}{\sigma\rho_2}\right)^{\frac{1}{2}} \qquad \dots \quad (10)$$

where $Z_0=C_0\rho_0$.

TABLE 2

$a=10^{-3}$ c.m., $n=10^6$. $\nu=.132$. $f_1=4.84\times10^{-3}\sqrt{N}$.

N	f_1	q	q^2	L	L^2	$\frac{1}{2}+\frac{9}{4a}\sqrt{\frac{2\nu}{\sigma}}$
128	.054	1.02	1.04	.66	.43	41.0
256	.075	2.05	4.10	.45	.20	30.0
512	.105	4.10	16.20	.24	.06	21.0
1024	.140	8.20	64.4	.12	.014	15.0
2048	.196	16.40	269.0	.06	.0036	11.0

Let

$$\chi=\tfrac{4}{3}\pi\rho_0\sigma a^3 n\left(\frac{1}{2}+\frac{9}{4a}\sqrt{\frac{2\nu}{\sigma}}\right)$$

$$\rho_2/\rho_0=1+\tfrac{4}{3}\pi a^3 n\frac{\rho_1}{\rho_0}L^2+\frac{\chi(1-L^2)}{\sigma\rho_0}+\frac{R_0 qL^2}{\sigma\rho_0}$$

or

$$\rho_2/\rho_0=1+\alpha+(\beta+\delta)/\sigma$$

where

$$d=\tfrac{4}{3}\pi a^3 n\frac{\rho_1}{\rho_0}L^2;\quad \beta=\chi(1-L^2)/\rho_0$$

$$\delta=R_0 qL^2/\rho_0.$$

TABLE 3

N	R_0/ρ_0	χ/ρ_0	d	β	δ	$(\beta+\delta)/\sigma$	ρ_2/ρ_0
128	2.48×10^3	1.28×10^2	1.33	$.73\times10^2$	1.11×10^3	1.50	3.83
256	2.53 ,,	1.94 ,,	.62	1.47 ,,	1.06 ,,	.80	2.42
512	2.66 ,,	2.60 ,,	.186	2.44 ,,	.66 ,,	.26	1.50
1024	2.76 ,,	3.8 ,,	.045	3.70 ,,	.31 ,,	.11	1.15
2048	2.90 ,,	5.55 ,,	.011	5.6 ,,	.20 ,,	.063	1.07

TABLE 4

N	$(1-\rho_2/\gamma\rho_0)^{\frac{1}{2}}$	R_1/ρ_0	$R_1/\sigma\rho_2$	Q	Z_2/Z_0	Reflection coeff.
128	1.67	1.36×10^3	.46	1.02−j.23	1.7 −j.38	24%
256	1.35	2.00 ,,	.54	1.04−j.27	1.32−j.34	14%
512	.98	2.44 ,,	.53	1.04−j.28	1.02−j.26	6.7%
1024	.90	2.66 ,,	.43	1.02−j.28	0.92−j.19	4.2%
2048	.79	2.87 ,,	.22	1.05−j.11	0.81−j.088	4.0%

TABLE 5

$a = 10^{-4}$ cm.; $n = 10^{9}$

N	f_1	q	L	$\frac{1}{2}+\frac{9}{4a}\sqrt{\frac{2\nu}{\sigma}}$	χ/ρ_0	R_0/ρ_0	ρ_2/ρ_0	$\left(\frac{\Gamma\rho_2}{\gamma\rho_0}\right)^{\frac{1}{2}}$	$R_1/\sigma\rho_2$
128	.0054	.0012	1	410	1.25×10^{3}	2.38×10^{5}	7.2	2.3	-2.3×10^{-3}
256	.0075	.0205	1	300	1.84 ,,	2.384 ,,	5.6	2.0	-4.31×10^{-3}
512	.0105	.0410	1	210	2.58 ,,	2.40 ,,	4.85	1.85	-7.00×10^{-3}
1024	.0104	.0820	1	150	3.65 ,,	2.41 ,,	4.48	1.78	-1.06×10^{-2}
2048	.0196	.164	1	110	5.34 ,,	2.42 ,,	4.43	1.76	-1.6×10^{-2}

TABLE 6

N	Q	Z_2/Z_0	Reflection coefficient
128	$1+j\,2.3\times10^{-3}$	$2.3(1+j2.3\times10^{-3})$	40%
256	$1+j\,4.30$,,	$2.0(1+j4.3$,,)	33%
512	$1+j\,7.00$,,	$1.85(1+j7.00$,,)	30%
1024	$1+j10.6$,,	$1.78(1+j10.6$,,)	28%
2048	$1+j16.0$,,	$1.76(1+j16.0$,,)	27%

Tables (2) to (4) give the values of the relevant quantities in the case of water particles of radius 10^{-3} cm. which represents the size of fog particles, while tables (5) and (6) do the same for finer drops. It will be noticed that the amplitude of vibration of the former kind in relation to that of air particles, represented by L is appreciable at low frequencies but it drops down at high frequencies. While much finer particles 10^{-4} cm. radius, have practically the same amplitude as that of air particles throughout the given range of frequencies. It will further be observed from equation (10) that R_1 contributes to the reactance of the particle-air medium in Z_2. For particles of 10^{-3} cm. radius R_1 has a positive value and the reactance is negative. While for finer particles 10^{-4} cm. radius, R_1 is negative and the added reactance is positive. This indicates that in the case of fine particles they contribute mostly towards inertia of the medium, while their resistance to compression predominates in the case of particles of 10^{-3} cm. radius.

The effective value of the acoustical resistance of the medium is greatly altered. For particles of 10^{-3} cm. radius it varies from 1.7 to 0.81 times that of air at different frequencies. In the case of finer particles the resistance varies from 2.3 to 1.76 times that of air in the same range of frequencies. The reflection coefficient varies from 24% to 4%, while in the case of finer particles the variation ranges from 40% to 27% over the same range of frequencies.

It will also be noted that on account of the smaller available volume for compression and the inertia, the velocity of sound in air containing suspensions, is greatly altered. The effective part taken by the various factors can be easily estimated from equation (9). Most important part is taken by the porosity P, the altered Γ and the effective density. The formula relating to Γ supposes that the suspensions remain at rest, while they actually oscillate, hence it does not represent the correct value of Γ. It will actually be less than that given by (3), because by their motion work of compression will raise the temperature more than that for stationery particles.

It has been pointed out in the early part of the paper that the *flow resistance* R′ can be substituted for the actual resistance due to oscillatory air current at low frequencies to a first approximation. At high frequency this approximation fails.

The present paper has given a picture of the phenomenon that takes place in the case of a gaseous medium containing suspensions; it is now possible to extend these results to the case of porous bodies, and the author proposes to do so in a subsequent paper.

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REFERENCES

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